RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4756

Further Methods for Advanced Mathematics (FP2)

Monday **16 JANUARY 2006** Morning 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions in Section A and **one** question from Section B.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

This question paper consists of 4 printed pages.

Section A (54 marks)

Answer all the questions

1 **(a)** A curve has polar equation $r = a\cos 3\theta$ for $-\frac{1}{2}\pi \leqslant \theta \leqslant \frac{1}{2}\pi$, where a is a positive constant.

(i) Sketch the curve, using a continuous line for sections where $r > 0$ and a broken line for sections where $r < 0$. [3]

(ii) Find the area enclosed by one of the loops. [5]

(b) Find the exact value of $\displaystyle\int_0^{\frac{3}{4}} \frac{1}{\sqrt{3-4x^2}}\,\mathrm{d}x$. [5]

(c) Use a trigonometric substitution to find $\displaystyle\int_0^{1} \frac{1}{(1+3x^2)^{\frac{3}{2}}}\,\mathrm{d}x$. [5]

2 In this question, θ is a real number with $0 < \theta < \frac{1}{6}\pi$, and $w = \frac{1}{2}\mathrm{e}^{3\mathrm{j}\theta}$.

(i) State the modulus and argument of each of the complex numbers

$$w, \quad w^* \quad \text{and} \quad \mathrm{j}w.$$

Illustrate these three complex numbers on an Argand diagram. [6]

(ii) Show that $(1+w)(1+w^*) = \frac{5}{4} + \cos 3\theta$. [4]

Infinite series C and S are defined by

$$C = \cos 2\theta - \tfrac{1}{2}\cos 5\theta + \tfrac{1}{4}\cos 8\theta - \tfrac{1}{8}\cos 11\theta + \dots,$$

$$S = \sin 2\theta - \tfrac{1}{2}\sin 5\theta + \tfrac{1}{4}\sin 8\theta - \tfrac{1}{8}\sin 11\theta + \dots.$$

(iii) Show that $C = \dfrac{4\cos 2\theta + 2\cos\theta}{5 + 4\cos 3\theta}$, and find a similar expression for S. [8]

3 The matrix $\mathbf{M} = \begin{pmatrix} 1 & 2 & 3 \\ -2 & -3 & 6 \\ 2 & 2 & -4 \end{pmatrix}$.

(i) Show that the characteristic equation for $\mathbf{M}$ is $\lambda^3 + 6\lambda^2 - 9\lambda - 14 = 0$. [3]

(ii) Show that –1 is an eigenvalue of $\mathbf{M}$, and find the other two eigenvalues. [3]

(iii) Find an eigenvector corresponding to the eigenvalue –1. [3]

(iv) Verify that $\begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 3 \\ -2 \end{pmatrix}$ are eigenvectors of $\mathbf{M}$. [3]

(v) Write down a matrix $\mathbf{P}$, and a diagonal matrix $\mathbf{D}$, such that $\mathbf{M}^3 = \mathbf{PDP}^{-1}$. [3]

(vi) Use the Cayley-Hamilton theorem to express $\mathbf{M}^{-1}$ in the form $a\mathbf{M}^2 + b\mathbf{M} + c\mathbf{I}$. [3]

Section B (18 marks)

Answer one question

Option 1: Hyperbolic functions

4 **(a)** Solve the equation

$$\sinh x + 4\cosh x = 8,$$

giving the answers in an exact logarithmic form. [6]

(b) Find the exact value of $\int_0^2 e^x \sinh x \, dx$. [4]

(c) **(i)** Differentiate $\operatorname{arsinh}\left(\frac{2}{3}x\right)$ with respect to x. [2]

(ii) Use integration by parts to show that $\int_0^2 \operatorname{arsinh}\left(\frac{2}{3}x\right) dx = 2\ln 3 - 1$. [6]

Option 2: Investigation of curves

This question requires the use of a graphical calculator.

5 A curve has equation $y = \dfrac{x^3 - k^3}{x^2 - 4}$, where k is a positive constant and $k \neq 2$.

(i) Find the equations of the three asymptotes. [3]

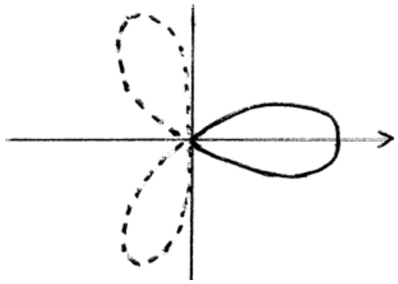
(ii) Use your graphical calculator to obtain rough sketches of the curve in the two separate cases $k < 2$ and $k > 2$. [4]

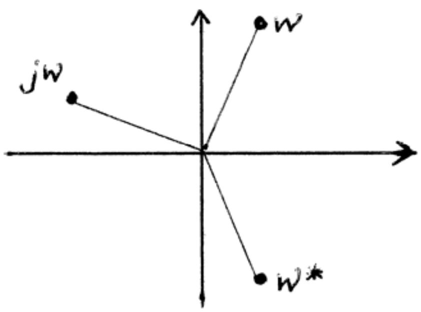
(iii) In the case $k < 2$, your sketch may not show clearly the shape of the curve near $x = 0$. Use calculus to show that the curve has a minimum point when $x = 0$. [5]

(iv) In the case $k > 2$, your sketch may not show clearly how the curve approaches its asymptote as $x \longrightarrow +\infty$. Show algebraically that the curve crosses this asymptote. [2]

(v) Use the results of parts **(iii)** and **(iv)** to produce more accurate sketches of the curve in the two separate cases $k < 2$ and $k > 2$. These sketches should indicate where the curve crosses the axes, and should show clearly how the curve approaches its asymptotes. The presence of stationary points should be clearly shown, but there is no need to find their coordinates. [4]

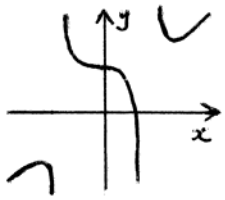
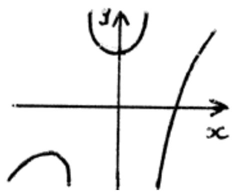
Mark Scheme 4756
January 2006

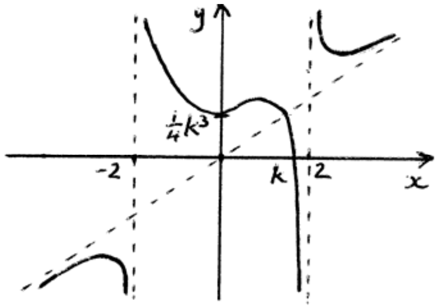
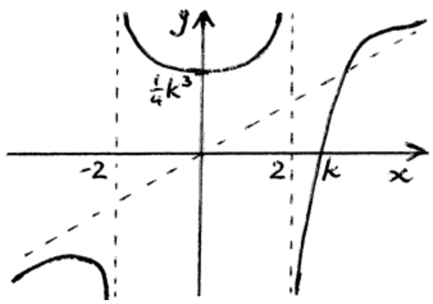
1(a)(i)		B1 B1 B1 **3**	For one loop in correct quadrant(s) For two more loops Continuous and broken lines *Dependent on previous B1B1*
(ii)	Area is $\int \frac{1}{2}r^2\,\mathrm{d}\theta = \int_{-\frac{1}{6}\pi}^{\frac{1}{6}\pi} \frac{1}{2}a^2\cos^2 3\theta\,\mathrm{d}\theta$ $= \int_{-\frac{1}{6}\pi}^{\frac{1}{6}\pi} \frac{1}{4}a^2(1+\cos 6\theta)\,\mathrm{d}\theta$ $= \left[\ \frac{1}{4}a^2(\theta + \frac{1}{6}\sin 6\theta)\ \right]_{-\frac{1}{6}\pi}^{\frac{1}{6}\pi}$ $= \frac{1}{12}\pi a^2$	M1 A1 M1 A1 B1 **5**	For $\int \cos^2 3\theta\,\mathrm{d}\theta$ For a correct integral expression including limits *(may be implied by later work)* For $\int \cos^2 3\theta\,\mathrm{d}\theta = \frac{1}{2}\theta + \frac{1}{12}\sin 6\theta$ Accept $0.262a^2$
(b)	$\int_0^{\frac{3}{4}} \frac{1}{\sqrt{3-4x^2}}\,\mathrm{d}x = \left[\ \frac{1}{2}\arcsin\left(\frac{2x}{\sqrt{3}}\right)\ \right]_0^{\frac{3}{4}}$ $= \frac{1}{2}\arcsin\left(\frac{3}{2\sqrt{3}}\right)$ $= \frac{1}{6}\pi$	M1 A1A1 M1 A1 **5**	For arcsin For $\frac{1}{2}$ and $\frac{2x}{\sqrt{3}}$ *Dependent on previous M1*
(c)	Putting $\sqrt{3}x = \tan\theta$ Integral is $\int_0^{\frac{1}{3}\pi} \frac{1}{\sec^3\theta}\left(\frac{\sec^2\theta}{\sqrt{3}}\right)\mathrm{d}\theta$ $= \int_0^{\frac{1}{3}\pi} \frac{\cos\theta}{\sqrt{3}}\,\mathrm{d}\theta = \left[\ \frac{\sin\theta}{\sqrt{3}}\ \right]_0^{\frac{1}{3}\pi}$ $= \frac{1}{2}$	M1 A1A1 M1 A1 **5**	For any tan substitution For $\frac{1}{(\sec^2\theta)^{\frac{3}{2}}}$ and $\frac{\sec^2\theta}{\sqrt{3}}$ Including limits of θ
	OR Putting $2x = \sqrt{3}\sin\theta$ — M1 A1 Integral is $\int_0^{\frac{1}{3}\pi} \frac{1}{2}\,\mathrm{d}\theta$ — A1 M1 $= \frac{1}{6}\pi$ — A1		For any sine substitution For $\int \frac{1}{2}\,\mathrm{d}\theta$ For changing to limits of θ *Dependent on previous M1*

2 (i)	$\lvert w \rvert = \frac{1}{2}$, $\arg w = 3\theta$ $\lvert w^* \rvert = \frac{1}{2}$, $\arg w^* = -3\theta$ $\lvert \mathrm{j}w \rvert = \frac{1}{2}$, $\arg \mathrm{j}w = 3\theta + \frac{1}{2}\pi$ 	B1 B1 ft B1B1 ft B2 **6**	w^* and $\mathrm{j}w$ in correct positions relative to their w in first quadrant Give B1 for at least two points in correct quadrants
(ii)	$(1+w)(1+w^*) = 1 + \frac{1}{2}\mathrm{e}^{3\mathrm{j}\theta} + \frac{1}{2}\mathrm{e}^{-3\mathrm{j}\theta} + (\frac{1}{2}\mathrm{e}^{3\mathrm{j}\theta})(\frac{1}{2}\mathrm{e}^{-3\mathrm{j}\theta})$ $= 1 + \frac{1}{2}(\cos 3\theta + \mathrm{j}\sin 3\theta) + \frac{1}{2}(\cos 3\theta - \mathrm{j}\sin 3\theta) + \frac{1}{4}$ $= \frac{5}{4} + \cos 3\theta$	M1 A1 M1 A1 (ag) **4**	for $w^* = \frac{1}{2}\mathrm{e}^{-3\mathrm{j}\theta}$ for $1 + \frac{1}{4}$ correctly obtained for $w = \frac{1}{2}(\cos 3\theta + \mathrm{j}\sin 3\theta)$ for $\cos 3\theta$ correctly obtained
(iii)	$C + \mathrm{j}S = \mathrm{e}^{2\mathrm{j}\theta} - \frac{1}{2}\mathrm{e}^{5\mathrm{j}\theta} + \frac{1}{4}\mathrm{e}^{8\mathrm{j}\theta} - \ldots$ $= \dfrac{\mathrm{e}^{2\mathrm{j}\theta}}{1 + \frac{1}{2}\mathrm{e}^{3\mathrm{j}\theta}}$ $= \dfrac{\mathrm{e}^{2\mathrm{j}\theta}(1 + \frac{1}{2}\mathrm{e}^{-3\mathrm{j}\theta})}{(1 + \frac{1}{2}\mathrm{e}^{3\mathrm{j}\theta})(1 + \frac{1}{2}\mathrm{e}^{-3\mathrm{j}\theta})}$ $= \dfrac{\mathrm{e}^{2\mathrm{j}\theta}(1 + \frac{1}{2}\mathrm{e}^{-3\mathrm{j}\theta})}{\frac{5}{4} + \cos 3\theta}$ $= \dfrac{\mathrm{e}^{2\mathrm{j}\theta} + \frac{1}{2}\mathrm{e}^{-\mathrm{j}\theta}}{\frac{5}{4} + \cos 3\theta}$ $\left(= \dfrac{4\mathrm{e}^{2\mathrm{j}\theta} + 2\mathrm{e}^{-\mathrm{j}\theta}}{5 + 4\cos 3\theta} \right)$ $C = \dfrac{4\cos 2\theta + 2\cos\theta}{5 + 4\cos 3\theta}$ $S = \dfrac{4\sin 2\theta - 2\sin\theta}{5 + 4\cos 3\theta}$	M1 M1 A1 M1 A1 M1 A1 (ag) A1 **8**	Obtaining a geometric series Summing an infinite geometric series Using complex conjugate of denom Equating real or imaginary parts Correctly obtained

3 (i)	$(1-\lambda)[\,(-3-\lambda)(-4-\lambda)-12\,]$ $-2[\,-2(-4-\lambda)-12\,]+3[\,-4-2(-3-\lambda)\,]=0$ $(1-\lambda)(\lambda^2+7\lambda)-2(2\lambda-4)+3(2\lambda+2)=0$ $\lambda^3+6\lambda^2-9\lambda-14=0$	M1 A1 A1 (ag) **3**	Evaluating $\det(\mathbf{M}-\lambda\mathbf{I})$ *Allow one omission and two sign errors* $\det(\mathbf{M}-\lambda\mathbf{I})$ correct Correctly obtained (=0 is required)
(ii)	When $\lambda=-1$, $-1+6+9-14=0$ $(\lambda+1)(\lambda^2+5\lambda-14)=0$ $(\lambda+1)(\lambda-2)(\lambda+7)=0$ Other eigenvalues are $2,\ -7$	B1 M1 A1 **3**	or showing that $(\lambda+1)$ is a factor, and deducing that -1 is a root for $(\lambda+1)\times$quadratic factor
(iii)	$x+2y+3z=-x$ $-2x-3y+6z=-y$ $2x+2y-4z=-z$ $z=0,\ x+y=0$ An eigenvector is $\begin{pmatrix}1\\-1\\0\end{pmatrix}$	M1 M1 A1 **3**	At least two equations Solving to obtain an eigenvector
	OR $\begin{pmatrix}2\\2\\3\end{pmatrix}\times\begin{pmatrix}-2\\-2\\6\end{pmatrix}=\begin{pmatrix}18\\-18\\0\end{pmatrix}$ M1 M1 A1		Appropriate vector product Evaluation of vector product
(iv)	$\mathbf{M}\begin{pmatrix}3\\0\\1\end{pmatrix}=\begin{pmatrix}6\\0\\2\end{pmatrix}=2\begin{pmatrix}3\\0\\1\end{pmatrix}$ $\mathbf{M}\begin{pmatrix}0\\3\\-2\end{pmatrix}=\begin{pmatrix}0\\-21\\14\end{pmatrix}=-7\begin{pmatrix}0\\3\\-2\end{pmatrix}$	M1 A1A1 **3**	Any method for verifying or finding an eigenvector
(v)	$\mathbf{P}=\begin{pmatrix}1&3&0\\-1&0&3\\0&1&-2\end{pmatrix}$ $\mathbf{D}=\begin{pmatrix}-1&0&0\\0&2&0\\0&0&-7\end{pmatrix}^3$ $=\begin{pmatrix}-1&0&0\\0&8&0\\0&0&-343\end{pmatrix}$	B1 ft M1 A1 ft **3**	seen or implied (ft) *(condone eigenvalues in wrong order)* Order must be consistent with **P** (when B1 has been awarded)
(vi)	By CHT, $\mathbf{M}^3+6\mathbf{M}^2-9\mathbf{M}-14\mathbf{I}=\mathbf{0}$ $\mathbf{M}^2+6\mathbf{M}-9\mathbf{I}-14\mathbf{M}^{-1}=\mathbf{0}$ $\mathbf{M}^{-1}=\frac{1}{14}\mathbf{M}^2+\frac{3}{7}\mathbf{M}-\frac{9}{14}\mathbf{I}$	B1 M1 A1 **3**	Condone omission of **I** Condone dividing by **M**

4 (a)	$\frac{1}{2}(e^x - e^{-x}) + 2(e^x + e^{-x}) = 8$ $5e^{2x} - 16e^x + 3 = 0$ $(5e^x - 1)(e^x - 3) = 0$ $e^x = \frac{1}{5},\ 3$ $x = -\ln 5,\quad \ln 3$	M1 M1 M1 A1A1 A1 ft **6**	Exponential form Quadratic in e^x Solving to obtain a value of e^x Exact logarithmic form from 2 positive values of e^x *Dependent on M3*
	OR $\sqrt{c^2 - 1} = 8 - 4c$ $15c^2 - 64c + 65 = 0$ M1 M1 $c = \frac{5}{3},\ \frac{13}{5}$ A1A1 $x = \pm\ln 3,\ \pm\ln 5$ M1 $x = \ln 3,\ -\ln 5$ A1		Obtaining quadratic in c (or s) $(15s^2 + 16s - 48 = 0)$ Solving to obtain a value of c (or s) or $s = \frac{4}{3},\ -\frac{12}{5}$ Logarithmic form (including $\pm$ if c) cao
(b)	$\int_0^2 \frac{1}{2}e^x(e^x - e^{-x})\,dx$ $= \left[\frac{1}{4}e^{2x} - \frac{1}{2}x\right]_0^2$ $= (\frac{1}{4}e^4 - 1) - (\frac{1}{4})$ $= \frac{1}{4}(e^4 - 5)$	M1 M1 A1 A1 **4**	Exponential form Integrating to obtain a multiple of e^{2x}
(c)(i)	$\dfrac{\frac{2}{3}}{\sqrt{1 + (\frac{2}{3}x)^2}} \quad \left(= \dfrac{2}{\sqrt{9 + 4x^2}} \right)$	B2 **2**	Give B1 for any non-zero multiple of this
(ii)	$\left[x\,\text{arsinh}(\frac{2}{3}x)\right]_0^2 - \int_0^2 \dfrac{2x}{\sqrt{9 + 4x^2}}\,dx$ $= \left[x\,\text{arsinh}(\frac{2}{3}x) - \frac{1}{2}\sqrt{9 + 4x^2}\right]_0^2$ $= \left(2\,\text{arsinh}(\frac{4}{3}) - \frac{5}{2}\right) - (-\frac{3}{2})$ $= 2\ln\left(\frac{4}{3} + \sqrt{1 + \frac{16}{9}}\right) - 1$ $= 2\ln 3 - 1$	M1 A1 ft B1 M1 M1 A1 (ag) **6**	Integration by parts applied to $\text{arsinh}(\frac{2}{3}x) \times 1$ for $\int \dfrac{x}{\sqrt{9 + 4x^2}}\,dx = \frac{1}{4}\sqrt{9 + 4x^2}$ Using both limits (provided both give non-zero values) Logarithmic form for arsinh (intermediate step required)

5 (i)	$x = 2$, $x = -2$ $y = x + \dfrac{4x - k^3}{x^2 - 4}$ Asymptote is $y = x$	B1 M1 A1 **3**	 Dividing out or B2 for $y = x$ stated
(ii)	  $k < 2$ $k > 2$	B1 B1 B1 B1 **4**	$k < 2$ for LH and RH sections for central section, with positive intercepts on both axes $k > 2$ for LH and central sections for RH section, crossing x-axis
(iii)	$\dfrac{dy}{dx} = \dfrac{(x^2-4)(3x^2) - (x^3 - k^3)(2x)}{(x^2-4)^2}$ $= \dfrac{x(2k^3 + x^3 - 12x)}{(x^2-4)^2}$ $\dfrac{dy}{dx} = 0$ when $x = 0$ When $x \approx 0$, $2k^3 + x^3 - 12x > 0$ $\dfrac{dy}{dx} < 0$ when $x < 0$, $\dfrac{dy}{dx} > 0$ when $x > 0$ Hence there is a minimum when $x = 0$	M1 A1 A1 (ag) M1 A1 (ag) **5**	Using quotient rule (or equivalent) Any correct form Correctly shown or evaluating $\dfrac{d^2y}{dx^2}$ when $x = 0$ or $\dfrac{d^2y}{dx^2} = \frac{1}{8}k^3 > 0$ when $x = 0$
(iv)	Curve crosses $y = x$ when $x^3 - k^3 = x(x^2 - 4)$ $x = \frac{1}{4}k^3$ So curve crosses this asymptote	M1 A1 (ag) **2**	

(v)	$k < 2$ 	B2	Asymptotes shown Intercepts $\frac{1}{4}k^3$ and k indicated Minimum on positive *y*-axis Maximum shown Give B1 for minimum and maximum on central section
	$k > 2$ 	B2 **4**	Asymptotes shown Intercepts $\frac{1}{4}k^3$ and k indicated Minimum on positive *y*-axis RH section crosses $y = x$ and approaches it from above Give B1 for RH section approaching both asymptotes correctly

4756: Further Methods for Advanced Mathematics (FP2)

General Comments

There was quite a range of performance on this paper. There were some really good scripts, with about 10% of candidates scoring more than 60 marks out of 72. On the other hand, about 20% of candidates scored less than 30 marks; many were clearly not ready to take the paper and found it to be a severe challenge. Some candidates appeared to run out of time, but this was usually a consequence of using very long and complicated methods in the integration questions.

In Section A, the work on the matrices topic (question 3) was of a much higher standard than that on calculus and complex numbers.

In Section B, almost every candidate chose the question on hyperbolic functions.

Comments on Individual Questions

1) This question, on polar equations and integration, was found to be quite difficult, especially part (c), The average mark was about 10 out of 18.

In part (a)(i), most candidates drew a curve of the correct shape with three loops, but the use of continuous and broken lines was usually incorrect. A common error was to use broken lines in the third and fourth quadrants, which corresponds to the domain $0 \le \theta \le \pi$ instead of the given $-\frac{1}{2}\pi \le \theta \le \frac{1}{2}\pi$.

In part (a)(ii), the calculation of the area was generally well understood, although the limits of integration were quite often incorrect. Most candidates realised that the integration of $\cos^2 3\theta$ required the use of a double angle formula, but the details were not always correct.

In part (b), most candidates recognised that this integral involved arcsin, and some were able to write down a completely correct result with little difficulty. The factor $\frac{1}{2}$ was often omitted, and $\frac{4x}{3}$ sometimes appeared instead of $\frac{2x}{\sqrt{3}}$.

In part (c), very many candidates did not make a tan substitution, and so were unable to make any progress. Some used the correct substitution and obtained an integral involving $\frac{\sec^2\theta}{\sec^3\theta}$ but failed to simplify this to $\cos\theta$ and complete the integration. Only a few obtained the correct answer.

2) This question, on complex numbers, was the worst answered, with an average mark of about 9 out of 18.

Some candidates sailed through part (i), but the majority made at least one slip, particularly with the arguments; quite a few gave the modulus of jw as $\frac{1}{2}\mathrm{j}$. Many appeared to be very uncertain about what was required, possibly because there was not a precise value of θ to work with.

The proof in part (ii) was handled well, and usually scored full marks. Those who started by writing $w^* = \frac{1}{2}\mathrm{e}^{-3\mathrm{j}\theta}$ had an easier time than those who went straight to $w^* = \frac{1}{2}(\cos 3\theta - \mathrm{j}\sin 3\theta)$.

In part (iii), most candidates knew that they should consider $C + \mathrm{j}S$, but many seemed to be unfamiliar with the methods required to progress beyond this, so the marks in this part were often low. Some who obtained the correct sum of the infinite series were unable to convert it into a form with a real denominator. However, there were some confident and efficient solutions from candidates who recognised the connection with part (ii) and then kept the numerator in exponential form.

3) This question, on matrices, was by far the best answered, with an average mark of about 14 out of 18. Most candidates displayed good algebraic and numerical skills.

In part (i) the characteristic equation was usually obtained correctly, by a great variety of methods. There was even some use of elementary row operations.

In part (ii), almost all candidates found the eigenvalues accurately.

Part (iii) was often answered well, although some candidates solved $(\mathbf{M}+\mathbf{I})\mathbf{x}=-\mathbf{x}$ or $(\mathbf{M}-\mathbf{I})\mathbf{x}=\mathbf{0}$ instead of $(\mathbf{M}+\mathbf{I})\mathbf{x}=\mathbf{0}$.

Most candidates were successful in part (iv). The simplest method was to transform the given vectors and recognise the images as multiples of the original vectors, but some used much longer methods, deriving the eigenvectors in the same way as in part (iii).

In part (v), most candidates knew that **P** was the matrix of eigenvectors, but many gave **D** as the diagonal matrix of eigenvalues instead of their cubes.

In part (vi), the Cayley-Hamilton theorem and its application were generally well understood. Sometimes **I** was omitted from the equations.

Section B

4) The average mark for this question, on hyperbolic functions, was about 10 out of 18.

In part (a), most candidates converted the equation to a quadratic in exponential form, with a substantial number obtaining the correct answers.

In part (b), those who wrote $\sinh x$ in exponential form were usually successful, although there were a few sign errors. Very many attempted to use integration by parts, which is not an appropriate method here.

In part (c), the general form of the derivative of $\operatorname{arsinh}\frac{2}{3}x$ was usually correct, although many had an incorrect numerical factor. Integration by parts was often applied correctly, but very few managed to produce a completely convincing derivation of the given answer. The first difficulty was the integration of *x* times their answer to part (i); many stopped at this point, and others obtained an incorrect numerical factor. The next problem was the derivation of the $2\ln 3$ term; $\operatorname{arsinh}\frac{4}{3}=\ln 3$ was often stated without any explanation.

5) There were fewer than ten attempts at this question on the investigation of graphs. There was some competent work in parts (i) to (iv), but no candidate scored any marks for the improved sketches in part (v).